Using Eye Models to Describe Ocular Wavefront Aberrations

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Eye models are a framework for thinking

Descartes was the first to create an optical model of the transparent structures in the eye refracting light to form an image of the world on the retinal surface.

His was the first step in a journey we are still traveling at WFC-2016:

- Understanding the eye’s optical flaws, their causes, and how to correct them.
- How the eye grows itself to match the world it lives in.
- How the brain integrates dissimilar retinal images from the left and right eyes.
- Understanding the principles of new design concepts for improving vision.

Descartes (1662)
Simplify with a “reduced eye” model

The reduced eye is the standard teaching tool used by students learning paraxial Gaussian optics and refractive errors of the eye.

The reduced eye is versatile enough to learn the basics of:

• defocus & astigmatism
• chromatic aberration
• spherical aberration
• higher-order aberrations
• image quality
Begin with the concept of optical path length (OPL)

Consider two points separated by physical distance \( d \).

\( A \quad 570\text{nm light oscillates 1 million times to travel 57cm} \quad B \)

\[ \text{Number of oscillations} = \text{frequency (cyc/sec)} \times \text{time(sec)} \]
\[ = \text{freq} \times \frac{d}{v} = \left(\frac{\text{freq}}{c}\right) \times d \times n = d \times n \times \frac{\lambda_0}{\lambda} \]

\[ \text{OPL} = d \times n = \text{# of oscillations} \times \lambda_0 \text{ = equiv. distance in a vacuum} \]
OPL concept defines the optically perfect eye

If all rays of light travel the *same optical distance* to get from a point source of light $P$, through the pupil, to arrive at a point on the retinal surface $P'$, then light traveling those paths will have oscillated the same number of times and will produce the *perfect image* of the point source.

This definition of perfection tells us how to measure imperfection as an OPL error.
Optical path errors in a reduced eye model

OPL for chief ray = [PO] + n*[OP']
OPL for example ray = [PQ] + n*[QP']

Optical path difference (OPD) may be associated with entrance pupil location A or with exit pupil location D.

For an aberrated ray, QP' is NOT the ray's actual path. It is the ideal, shortest-possible path.
A wavefront is the locus of points of equal optical distance from a point source. The wavefront may be located by tracing all rays the same optical distance from P.

OPD represents a difference in propagation time from P to P', which causes temporal phase errors resulting in destructive interference & image degradation.

OPD = n*[CD] = “Wavefront Error” measured along the ideal path

Emerging wavefront is defined by equal OPLs: [PQ]+n[QC] = [PO]+n[OE']

Deviation of the wavefront from a reference sphere centered on P' indicates the presence of aberrations.

Reference sphere centered on the Gaussian image P' of P and passing through E', the center of the exit pupil.
Alternative formulation of optical path difference (OPD)

Rather than measuring OPL along the ideal path, ray-tracing programs typically compute OPL along the actual path of the ray predicted by Snell’s Law: \( n \sin \theta = n' \sin \theta' \). This convention approximates theory.

Theory: \( \text{OPD} = n[D_1D_2] \)
Practice: \( \text{OPD} = n[C_1C_2] \)

Example: Zemax optical design program computes OPD as:
\[
[PQ] + n[QC_2] - [PO] - n[OE'] = n[C_1C_2]
\]
A better formula for numerical calculations

To avoid astronomically large object distances, measure OPLs from incident reference sphere to exit reference sphere. Since \([PO]=[PB]\), we can start measuring at point B.

Theory: \(\text{OPL} = BQ + n[QD]\)
Practice: \(\text{OPL} = BQ + n[QC]\)

Reference sphere centered on P and passing through O, where the chief ray intersects the refracting surface.

Reference sphere centered on P', the Gaussian image of P and passing through E', the center of the exit pupil.
Inferring wavefront aberrations from surface aberrations

- Wavefront aberrations are measured by the separation between a wavefront of light and a perfect (i.e. spherical) reference wavefront.

- Similarly, *surface aberrations* are measured by the separation between the physical refracting surface and a perfect (i.e. Cartesian oval) reference surface.

- Therefore, we can construct an eye model with specific aberrations by modulating the shape of the refracting surface.

- A simple method results from examining the link between OPD and surface sag relative to the Cartesian oval.
Linking surface sag to wavefront aberrations

For the aberrated refracting surface, \( \text{OPD} = [TS] + n[SP] - n[OP] \)
but \( [OP] = [TQ] + n[QP] \) (by definition of the Cartesian oval).
Therefore, \( \text{OPD} = [QS] - n([QP] - [SP]) \approx [QS] - n[QV] \)
So, \( \text{OPD} \approx [QS](1 - \text{ncos}(\phi)) = \text{surface sag} \times (1 - \text{ncos}(\phi)) \)
Customized model of an eye with known aberrations

\[ S(x,y) = \text{refracting surface} \]

\[ Q(x,y) = \text{the perfect refracting surface for distant objects (Cartesian ellipse)} \]

\[ \text{OPD} = \text{sag}(1-n\cos\phi) \]

\[ \frac{\text{OPD}}{(1-n\cos\phi)} = \text{sag} \]

\[ n = \text{refractive index of ocular medium} \]

\[ \text{Free-form refracting surface} \]

\[ \text{Aberration map} \]

**Method:**

1) Start with the perfect surface (Cartesian ellipse)
2) For every point on the surface, introduce the amount of sag needed to reproduce the desired OPD.
Resulting surface reproduces the measured aberrations.

\[ \text{Surface } S(x,y) = Q(x,y) + \frac{\text{OPD}(x,y)}{1 - n\cos\phi(x,y)} \]
Wide-angle eye-models

The preceding developments were for axial point sources. To extend eye models to a wider field of view, move the source off the optical axis and repeat the optical analysis.

Even a rotationally symmetric surface appears astigmatic when light rays strike the surface obliquely (θ >> 0°). In this case, calculation of the Gaussian image point P’ is aided by the use of Coddington’s equations for oblique astigmatism.

Coddington’s equations for spherical surface*

\[
P_{\text{Sagittal}} = \frac{n'\cos i' - n\cos i}{r} \\
P_{\text{Tangential}} = \frac{n'\cos i' - n\cos i}{r\cos^2 i'} \\
J = P_{\text{Sagittal}} - P_{\text{Tangential}}
\]

*For toric surfaces, use generalized Coddington’s equations (GCE)
The elliptical pupil problem

- When viewed from the side, a circular pupil appears elliptical.
- Ray tracing programs (e.g. Zemax) typically report OPDs for a square array of sample locations in the circular entrance pupil, independent of field location of the source.
- That square array of sample locations appears compressed into a rectangular array when viewed from the side.
- These changes in pupil shape and geometry of sampling locations must be taken into account when interpreting aberrations (e.g. Zernike coefficients) reported by ray-tracing programs.

Sample points for ray tracing through a circular entrance pupil

Apparent sample points for elliptical entrance pupil
Examples of potential mis-interpretations of wavefront aberration maps (and Zernike coeffs.) reported by Zemax

Reported astigmatism in a circular pupil

Appears as defocus in an elliptical pupil

Reported defocus in a circular pupil

Appears as astigmatism in an elliptical pupil

Wavefront errors over the elliptical pupil determines peripheral retinal image quality or the prescribed correcting lens.
Variations on the reduced eye model

The shape of the refracting surface of the reduced eye model can be modified to study various monochromatic aberrations.

- Cartesian oval for perfect imaging (no aberrations on-axis):
- Toroidal surface to induce axial astigmatism:
  - \[ ax^2 + by^2 = 2Rz - pz^2 \] (R=apical radius, p = conic shape factor)
- Aspheric conic surface to manipulate spherical aberration:
- Off-axis imaging by aspheric surface to induce oblique astigmatism:
- Free-form surface to model individual eyes with known aberrations:
- Off-axis imaging by a toroidal surface: interaction of axial and oblique astigmatism:
  - Tao Liu – WFC 2016
Variations on the basic eye model

• Use a dispersive ocular medium (refractive index varies with wavelength) to study longitudinal chromatic aberration (LCA) and its interaction with monochromatic aberrations.

• Decenter the pupil to study transverse chromatic aberration (TCA) and its interaction with LCA and monochromatic aberrations.

• Use the wavefront aberration function derived from an eye model to compute retinal images of objects and to quantify image quality.
Law of Parsimony: use the simplest model that works!

With the right parameters, a wide-angle eye model with homogeneous elements\(^1\) can match the wavefront aberrations of a gradient index (GRIN) model\(^2\).

Law of Parsimony: use the simplest model that works!

With the right parameters, a wide-angle reduced-eye model with a single refracting element\(^1\) can match the wavefront aberrations of a multi-surface model\(^2\)

For wide-angle models of the eye:

1. Sophisticated GRIN eye models can be mimicked by simpler models with uniform refractive index.

2. Eye models with uniform refractive index can be mimicked by even simpler models with a single refracting surface.

3. Therefore, it is logical to conclude that even sophisticated GRIN models can be mimicked by a reduced-eye model.

Despite their extreme simplicity, reduced-eye models are a useful framework for thinking and learning about visual optics.
The end